OBSERVATIONS OF THE COMET OF 1807.

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THE first time I observed this comet was on the twenty fifth day of September 1807, at seven o'clock in the evening, near the foot of the constellation *Virgo*. I did not make any accurate estimate of its place before the eighth of October, when I commenced a series of observations, by measuring its distance from several of the fixed stars, by a circular instrument of reflection of Borda's construction, and, to diminish the unavoidable errors of the observations, I generally measured ten or twelve distances from each star. This method was made use of until the seventeenth day of December, when the comet ceased to be visible to the naked eye. Not having any instrument proper to continue the observations, or even to keep sight of the comet for a much longer time, I applied to the Reverend Doctor Prince, who was so obliging as to fix several cross wires, at equal distances from each other, in the diaphragm of an excellent night-glass, having a vertical and horizontal motion in a stand; and by placing the wires nearly perpendicular to the horizon, we were enabled to estimate the difference of altitude and azimuth between the comet and any fixed star, near which it passed, more accurately than we otherwise could have done, though not with that degree of accuracy we could have wished. However, the observations, imperfect as they were, answered the valuable purpose of proving, that the elements of the orbit, calculated from the observations made with the circular instrument of reflection, gave the place of the comet at the period of its disappearance within the limits of the errors of the observations; as will be perceived by the last ob-

servation of this kind, that we made, which will be given at the end of this memoir.

The apparent motion of the comet was nearly in a great circle at the average rate of about a degree per day. It passed in succession through the constellations *Mons Mænalus*, *Serpens*, *Hercules*, *Lyra*, *Cygnus*, and *Lacerta*, and on the thirtieth day of January 1808 was near the extremity of the right hand of *Andromeda*. It had then the appearance of a *Nebula*, and was so faint, as to be hardly visible with the assistance of the night-glass. After this time I was prevented by indisposition from seeking for it.

In reducing the observations, made with the circular instrument, allowance was made for the variations of the distances during the time elapsed between observing the different stars, so as to make the distances correspond to the same moment of time. This correction being small was calculated with sufficient accuracy by taking a proportional part of the observed daily increase or decrease of distance. Allowance was made for refraction by adding to the observed distances a small correction, calculated by Shepherd's "*Tables* "*for correcting the apparent distance of the Moon and Stars, etc.*" by taking twenty times the correction in the column "*Var,*" corresponding to the observed distances and altitudes ;* this being the value for the mean temperature and density of the air. The Parallax was so small, that it was neglected. The Aberration, which sometimes exceeded a minute, was found and applied to the calculated longitudes and latitudes made use of in the last process for finding the elements of the orbit. The longitudes were always counted

* This tabular correction is equal to the variation of the apparent distance of any two heavenly bodies corresponding to a change of 20 degrees of Fahrenheit's thermometer, and is nearly one twentieth part of the correction arising from the whole refraction at the mean temperature, as may be easily proved by Dr. Bradley's rule for calculating the variation.

from the mean equinox. The right ascensions and declinations of the stars, made use of in reducing the observations, were taken from Zach's catalogue, published in Vince's Astronomy, and from Maskelyne's, published in his " Requisite Tables."

Early in November I began to calculate the elements of the orbit by the method given by La Place, in vol. i. book ii. § 37 of the " Mecanique Celeste." For this purpose I made use of all the observations from the first appearance of the comet to the sixth of November, as they are given in the table at the end of this article. To render this part of the calculation more simple, and to avoid divisions by fractions of a day, I reduced all the observations to seven o'clock in the evening, by applying to the observed places the corrections for the motion of the comet between the time of observation and seven o'clock. This correction was found sufficiently near for this purpose, by taking a proportional part of the daily variation of the comet in longitude and latitude. The observations having been thus prepared, I combined them together in ten different examples, each one containing five observations. By the mean of these calculations I found, for a first approximation, that the time of passing the perihelion was September 20, 1807, and the perihelion distance 0·65636, the mean distance of the sun from the earth being estimated as unity.

These elements I afterward corrected by the method explained by La Place in the same section of his work. For this purpose I made choice of the observations of October 8, October 23, and November 6. From these I found, for a second approximation, that the corrected time of passing the perihelion was September 19d. 7h. mean time at Salem, and the perihelion distance 0·6645. These elements, with the longitude of the node, the longitude of the perihelion, and the inclination of the orbit to the ecliptic corresponding, were published in the Salem Gazette of the tenth of November 1807; in which it was

mentioned, that these elements were very different from those of any of the known comets, published by De La Lande in his astronomy.

For a third approximation I combined the observations of October and November in ten examples similar to the last, and thus procured twenty equations like those given by La Place in vol. i. pag. 229, "Mecanique Celeste." From the mean of these the corrected elements were found as follow.

Perihelion distance	D=0·6485.
Time of passing the perihelion . .	T=Sep. 18d. 12h.
Place of the perihelion counted on the orbit of the comet . . .	P=9s. 0° 53′ 15″
Longitude of the ascending node .	N=8 26 36 29
Inclination of the orbit to the ecliptic .	I=63 13 31.

The geocentric longitudes and latitudes calculated by these elements were found in general not to differ more than five minutes from the observations in September, October, and November. To obtain a greater degree of accuracy I made use of the following method.

I supposed the corrections to be applied to the preceding elements to be represented by $0{,}002.d$, $0{,}2.t$, $10'.p$, $-10'.n$, $-10'.i$, which would make the corrected elements respectively as follow.

$$D + 0{,}002 \cdot d$$
$$T + 0{,}2 \cdot t$$
$$P + 10' \cdot p$$
$$N - 10' \cdot n$$
$$I - 10' \cdot i$$

and then calculated the values of d, t, p, n, and i, in the following manner.

First supposition. Making use of the elements D, T, P, N, and I, I calculated to seconds by Taylor's logarithms, the geocentric longitude and latitude of the comet for each of the observations contained in the following table from October 8 to December 17. Denoting

any one of these longitudes or latitudes by L' and the corresponding observed longitude or latitude by L, I put $L—L'=l$.

Second Supposition. Making use of all the preceding elements excepting D, and increasing that by the quantity 0·002, I recalculated the same geocentric longitude or latitude and denoted it by L'', and put $L'—L''=l'$.

Third Supposition. Making use of all the elements of the first supposition excepting T and increasing that by two tenths of a day, I recalculated the same geocentric longitude or latitude and denoted it by L''', and put $L'—L'''=l''$.

Fourth Supposition. Making use of all the elements of the first supposition excepting P and increasing that by ten minutes, I recalculated the same geocentric longitude or latitude and denoted it by L^{iv} and then put $L'—L^{iv}=l'''$.

Fifth Supposition. Making use of all the elements of the first supposition excepting N and decreasing that by ten minutes, I recalculated the same geocentric longitude or latitude and denoted it by L^{v} and then put $L'—L^{v}=l^{iv}$.

Sixth Supposition. Making use of all the elements of the first supposition excepting I and decreasing that by ten minutes, I recalculated the same geocentric longitude or latitude and denoted it by L^{vi} and then put $L'—L^{vi}=l^{v}$.

Each of the observed longitudes and latitudes by this means furnished an equation of the following form between the sought quantities d, t, p, n, i, upon the supposition, that the small variations of the calculated longitudes and latitudes are exactly proportional to the corresponding variations of the elements of the orbit.

$$o=l' \cdot d+l'' \cdot t+l''' \cdot p+l^{iv} \cdot n+l^{v}\, i+l.$$

So that by three observations of the comet we should procure six

equations, which is one more than is necessary for finding the values of d, t, p, n, i, and determining the elements of the orbit.

Now it is evident that if the comet moved exactly in a parabola, and the observations were perfectly correct, all these equations would concur in giving nearly* the same values of d, t, p, n, and i, but as errors arising from those sources are unavoidable, the substitution of the correct values of d, t, p, n, and i, in the equations will not generally make the second members equal to o; but will render them equal to the combined effect of the errors of the observations and of the parabolic hypothesis, and if we denote these errors by $x^{(1)}$, $x^{(2)}$, $x^{(3)}$, etc.† the equations will become of this form.‡

$$x^{(r)}=l'\cdot d+l''\cdot t+l'''\cdot p+l^{iv}\cdot n+l^{v}\cdot i+l.$$

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* The variations of the calculated latitudes and longitudes are not *accurately* proportional to the corresponding variations of the elements of the orbit, therefore the equations will not give *exactly* the same values of d, t, p, n, and i.

† It may be necessary to observe, that the figures annexed to x do not signify exponents of the powers of x, as the quantities $x^{(1)}$, $x^{(2)}$, etc. are wholly independent of each other.

‡ If the comet, during its appearance, had described so great an arch of its orbit, as to render it sensibly different from a parabola, we might calculate the elliptical elements by introducing another term into the above equation. For if D be the perihelion distance of the comet, U its angular distance from the perihelion, calculated for the time of any of the observations by the parabolic hypothesis, $U+x$ the same angular distance calculated in the elliptical hypothesis, supposing the perihelion distance of the comet, divided by its mean distance from the sun, to be E, we should have by the rules, given by La Place in the abovementioned work,

$$\text{Sine } x=\tfrac{1}{10}\,E\cdot\text{tang.}\,\tfrac{1}{2}U\cdot\left\{4-3\cdot\overline{\cos.\tfrac{1}{2}U}|^{2}-6\cdot\overline{\cos.\tfrac{1}{2}U}|^{4}\right\}$$

and the radius vector in the elliptical hypothesis would be

$$\frac{D}{\overline{\cos.\tfrac{1}{2}(U+x)}|^{2}}\cdot\left\{1-\frac{E}{2}\cdot\left(\text{tang.}\,\frac{U+x}{2}\right)^{2}\right\}$$

By

In this manner I calculated the equations, resulting from the twenty eight observations, made from October 8 to December 17, and obtained the following system of equations A, of which the first twenty eight were deduced from the observed longitudes and the rest from the observed latitudes, the equations being arranged according to the times of observation, and the errors in the observed longitudes counted in the same order, being denoted by $x^{(1)}$, $x^{(2)} \ldots x^{(28)}$, and the errors in the observed latitudes by $x^{(29)}$, $x^{(30)} \ldots x^{(56)}$.

$$x^{(1)} = -315 \cdot d + 2 \cdot t + 129 \cdot p + 239 \cdot n - 234 \cdot i + 38$$
$$x^{(2)} = -317 \cdot d - 1 \cdot t + 138 \cdot p + 242\ n - 248 \cdot i - 88$$
$$x^{(3)} = -319 \cdot d - 4 \cdot t + 145 \cdot p + 244\ n - 262 \cdot i - 98$$
$$x^{(4)} = -323 \cdot d - 8 \cdot t + 151 \cdot p + 246\ n - 279 \cdot i - 84$$
$$x^{(5)} = -330 \cdot d + 2 \cdot t + 198 \cdot p + 249\ n - 412 \cdot i + 21$$
$$x^{(6)} = -329 \cdot d + 14 \cdot t + 207 \cdot p + 246\ n - 448 \cdot i - 76$$
$$x^{(7)} = -327 \cdot d + 17 \cdot t + 208 \cdot p + 242\ n - 467 \cdot i + 16$$
$$x^{(8)} = -324 \cdot d + 26 \cdot t + 212 \cdot p + 240\ n - 485 \cdot i - 164$$
$$x^{(9)} = -321 \cdot d + 31 \cdot t + 213 \cdot p + 235\ n - 505 \cdot i - 116$$

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By these rules we might calculate the true anomaly and distance of the comet from the sun, and thence deduce the geocentric longitude and latitude for each of the observations, supposing the elements D, T, P, N, I, to be the same, as in the first supposition, and E to be a very small quantity, as for example $\frac{1}{100}$. Then, denoting any one of these longitudes or latitudes by L^{vii}, and putting $L' - L^{vii} = l^{vi}$ and $E = \frac{e}{100}$, we must add to the above value of $x^{(r)}$ the term $l^{vi}\, e$. By this means the system of equations A, B, C, etc. would contain terms depending on e, and it would be necessary to make another assumption similar to the four first, made use of in the above calculations, which would furnish another equation, by means of which e might be exterminated, and the equation reduced to the same form as the system F, from which the value of p might be found as above. The observations of this comet accorded so well with a parabolic hypothesis, that the differences between the calculated and true places were within the limits of the errors of the observations; therefore I did not attempt to calculate the elliptical orbit.

$$\begin{array}{lrrrrrr}
 & '' & '' & '' & '' & '' & '' \\
x^{(10)} = & -318\cdot d & +40\cdot t & +215\cdot p & +231\; n & -526\cdot i & -\;71 \\
x^{(11)} = & -314\cdot d & +48\cdot t & +215\cdot p & +225\; n & -547\cdot i & +\;86 \\
x^{(12)} = & -308\cdot d & +57\cdot t & +214\cdot p & +218\; n & -567\cdot i & -\;278 \\
x^{(13)} = & -304\cdot d & +67\cdot t & +213\cdot p & +211\; n & -589\cdot i & -\;158 \\
x^{(14)} = & -199\cdot d & +202\cdot t & +157\cdot p & +108\cdot n & -789\cdot i & -\;413 \\
x^{(15)} = & -140\cdot d & +262\cdot t & +119\cdot p & +59\cdot n & -851\cdot i & +\;385 \\
x^{(16)} = & -122\cdot d & +280\cdot t & +99\cdot p & +37\cdot n & -876\cdot i & +\;120 \\
x^{(17)} = & -95\cdot d & +305\cdot t & +81\cdot p & +16\cdot n & -895\cdot i & -\;304 \\
x^{(18)} = & -14\cdot d & +378\cdot t & +12\cdot p & -55\cdot n & -955\cdot i & +1081 \\
x^{(19)} = & 47\cdot d & +429\cdot t & -40\cdot p & -105\cdot n & -987\cdot i & +\;775 \\
x^{(20)} = & 187\cdot d & +533\cdot t & -165\cdot p & -217\cdot n & -1033\cdot i & +\;263 \\
x^{(21)} = & 267\cdot d & +588\cdot t & -240\cdot p & -280\cdot n & -1045\cdot i & +\;164 \\
x^{(22)} = & 306\cdot d & +613\cdot t & -282\cdot p & -314\cdot n & -1048\cdot i & +\;757 \\
x^{(23)} = & 509\cdot d & +726\cdot t & -483\cdot p & -464\cdot n & -1020\cdot i & +\;204 \\
x^{(24)} = & 542\cdot d & +742\cdot t & -520\cdot p & -489\cdot n & -1007\cdot i & +\;238 \\
x^{(25)} = & 582\cdot d & +761\cdot t & -562\cdot p & -517\cdot n & -992\cdot i & +\;614 \\
x^{(26)} = & 690\cdot d & +804\cdot t & -676\cdot p & -587\cdot n & -929\cdot i & +\;47 \\
x^{(27)} = & 724\cdot d & +813\cdot t & -713\cdot p & -609\cdot n & -905\cdot i & +1628 \\
x^{(28)} = & 944\cdot d & +789\cdot t & -1006\cdot p & -716\cdot n & -496\cdot i & +\;69 \\
x^{(29)} = & 226\cdot d & +957\cdot t & -336\cdot p & -214\cdot n & +43\cdot i & -\;180 \\
x^{(30)} = & 237\cdot d & +950\cdot t & -337\cdot p & -212\cdot n & +41\cdot i & -\;235 \\
x^{(31)} = & 247\cdot d & +945\cdot t & -339\cdot p & -211\cdot n & +39\cdot i & -\;301 \\
x^{(32)} = & 258\cdot d & +933\cdot t & -341\cdot p & -209\cdot n & +38\cdot i & -\;143 \\
x^{(33)} = & 333\cdot d & +865\cdot t & -361\cdot p & -208\cdot n & +28\cdot i & -\;262 \\
x^{(34)} = & 349\cdot d & +849\cdot t & -367\cdot p & -210\cdot n & +27\cdot i & -\;197 \\
x^{(35)} = & 359\cdot d & +839\cdot t & -372\cdot p & -211\cdot n & +27\cdot i & -\;386 \\
x^{(36)} = & 366\cdot d & +831\cdot t & -374\cdot p & -212\cdot n & +27\cdot i & -\;180 \\
x^{(37)} = & 375\cdot d & +824\cdot t & -378\cdot p & -213\cdot n & +28\cdot i & -\;98 \\
x^{(38)} = & 383\cdot d & +814\cdot t & -382\cdot p & -215\cdot n & +29\cdot i & -\;127 \\
x^{(39)} = & 390\cdot d & +804\cdot t & -386\cdot p & -217\cdot n & +28\cdot i & -\;92 \\
x^{(40)} = & 397\cdot d & +794\cdot t & -391\cdot p & -220\cdot n & +31\cdot i & +\;74
\end{array} \qquad \text{(A)}$$

$$\begin{array}{llllll} '' & '' & '' & '' & '' & '' \\ x^{(41)}=403\cdot d & +784\cdot t & -395\cdot p & -223\cdot n & +\ 32\cdot i & -108 \\ x^{(42)}=463\cdot d & +702\cdot t & -435\cdot p & -241\cdot n & +\ 74\cdot i & +244 \\ x^{(43)}=476\cdot d & +671\cdot t & -447\cdot p & -245\cdot n & +\ 97\cdot i & -294 \\ x^{(44)}=478\cdot d & +659\cdot t & -451\cdot p & -246\cdot n & +106\cdot i & -197 \\ x^{(45)}=483\cdot d & +651\cdot t & -454\cdot p & -246\cdot n & +115\cdot i & +413 \\ x^{(46)}=491\cdot d & +617\cdot t & -464\cdot p & -246\cdot n & +146\cdot i & -803 \\ x^{(47)}=493\cdot d & +593\cdot t & -468\cdot p & -242\cdot n & +170\cdot i & -200 \\ x^{(48)}=488\cdot d & +541\cdot t & -468\cdot p & -229\cdot n & +223\cdot i & -177 \\ x^{(49)}=484\cdot d & +516\cdot t & -466\cdot p & -220\cdot n & +252\cdot i & +287 \\ x^{(50)}=477\cdot d & +500\cdot t & -464\cdot p & -215\cdot n & +267\cdot i & +424 \\ x^{(51)}=447\cdot d & +428\cdot t & -440\cdot p & -176\cdot n & +342\cdot i & +565 \\ x^{(52)}=436\cdot d & +412\cdot t & -434\cdot p & -167\cdot n & +355\cdot i & +101 \\ x^{(53)}=427\cdot d & +397\cdot t & -426\cdot p & -157\cdot n & +370\cdot i & -143 \\ x^{(54)}=395\cdot d & +351\cdot t & -398\cdot p & -123\cdot n & +412\cdot i & -589 \\ x^{(55)}=384\cdot d & +338\cdot t & -387\cdot p & -111\cdot n & +425\cdot i & -1082 \\ x^{(56)}=206\cdot d & +167\cdot t & -219\cdot p & +\ 56\cdot n & +539\cdot i & -229 \end{array}$$

Having obtained these equations it remained to deduce from them the values of the unknown quantities d, t, p, n, and i. The method, I thought most advisable to pursue for this purpose, is founded on the following assumed principles.

1. That the sum of all the positive errors in the calculated longitudes should be equal to the sum of the negative ones. That is in symbols

$$x^{(1)}+x^{(2)}+x^{(3)}\ \ldots\ +x^{(28)}=0.$$

2. That the sum of all the positive errors in the calculated latitudes should be equal to the sum of the negative ones. That is,

$$x^{(29)}+x^{(30)}+x^{(31)}\ \ldots\ +x^{(56)}=0.$$

3. That the sum of all the positive errors in the observed longitudes in the observations made in October should be equal to the sum of the negative ones. That is,

$$x^{(1)}+x^{(2)}+x^{(3)}\ \ldots\ x^{(13)}=0.$$

4. That the sum of the positive errors in the observed latitudes the observations made in October should be equal to the sum of th negative ones. That is,

$$x(^{29})+x(^{30}+x(^{31}) \ldots x(^{41})=0.$$

5. The four preceding conditions having been fulfilled, the fifth that the sum of all the errors in the longitudes and latitudes taken positively should be a *minimum*.

In applying the four first of these conditions to the system of equations A, they furnished four equations between d, t, p, n, i, by means of which I exterminated d, t, n, i from the equations A. Thus the equations $x(^1)+x(^2)+x(^3) \ldots +x(^{56})=0$, found by adding together the equations $x(^1)+x(^2) \ldots +x(^{28}=0$ and $x(^{28})+x(^{29}) \ldots +x(^{56})=0$,* gives for the sum of all the equations A,

$0=11030 \cdot d+27248 \cdot t—12941 \cdot p—6648 \cdot n—15086 \cdot i+742$, which by dividing by—27248,

$$0=—0{\cdot}40480 \cdot d—t+0{\cdot}47493 \cdot p+0{\cdot}24398 \cdot n +0{\cdot}55366 \cdot i—0{\cdot}02723. \qquad (1)$$

By multiplying this equation successively by the coefficients of t in the equations A, I obtained a system of equations B, in which the coefficients of t were equal and of a contrary sign to those of the system A, and by adding together the corresponding equations of the systems A and B, I obtained the equations C independent of t, of this form :

$$\left.\begin{array}{l} x(^1)=—316 \cdot d+130 \cdot p+239 \cdot n—233 \cdot i+38 \\ x(^2)=—317 \cdot d+138 \cdot p+242 \cdot n—249 \cdot i—88 \\ \text{\&c. \&c.} \end{array}\right\} \quad (C)$$

The sum of the first twenty eight of these equations gives, in putting $x(^1)+x(^2) \ldots +x(^{28})=0$,

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* I took the sum of the two equations instead of the second of them from the circumstance of having previously calculated the sum of all the equations A.

$$0=-3368\cdot d+2284\cdot p+1011\cdot n-14683\cdot i+4424,$$

and by dividing by 14683

$$0=-0{\cdot}22944\cdot d+0{\cdot}15555\cdot p+0{\cdot}06885\cdot n-i+0{\cdot}30130 \quad (2).$$

By multiplying this equation successively by the coefficients of i in the equations C, and adding these new equations to the corresponding ones of the system C, I obtained the system of equations D independent of t and i of this form :

$$\left.\begin{aligned} x(^1)&=-263\cdot d+94\cdot p+223\cdot n-\ \ 32 \\ x(^2)&=-260\cdot d+99\cdot p+225\cdot n-163 \\ &\&c.\ \&c. \end{aligned}\right\} \quad (D)$$

The sum of the first thirteen of these equations gives in putting $x(^1)+x(^2)\ldots+x(^{13})=0$,

$$0=-3027\cdot d+1757\cdot p+2766\cdot n-2609,$$

and by dividing by 3027

$$0=-d+0{\cdot}58044\cdot p+0{\cdot}91378\cdot n-0{\cdot}86191. \quad (3)$$

By multiplying this equation successively by the coefficients of d in the equations D, and adding these new equations to the corresponding ones of the system D, I obtained the equations E, independent of d, t, and i.

$$\left.\begin{aligned} x(^1)&=-59\cdot p-17\cdot n+195 \\ x(^2)&=-52\cdot p-13\cdot n+\ \ 61 \\ &\&c.\ \&c. \end{aligned}\right\} \quad (E)$$

By adding these equations from the 29th to the 41st, and putting $x(^{29})+x(^{30})\ldots+x(^{41})=0$, I obtained this equation

$$0=580\cdot p-1164\cdot n+941,$$

and by dividing by 1164

$$0=0{\cdot}49828\cdot p-n+0{\cdot}80842. \quad (4)$$

This equation being multiplied successively by the coefficients of n in the equations E, and added to the corresponding ones of that system, produced the equations F.

$$\begin{array}{l} x(^1)=-67\cdot p+181 \\ x(^2)=-58\cdot p+\ 50 \\ x(^3)=-51\cdot p+\ 36 \\ x(^4)=-49\cdot p+\ 45 \\ \text{\&c. \&c.} \end{array} \qquad \text{(F)}$$

These equations are of the same form as those given by La Place in vol. ii, pag. 135 of his "Mecanique Celeste," and the value of p, which will render the sum of the errors taken positively a *minimum*, may be found by the method explained in that part of his work. In the first place the coefficients of the term p must be rendered positive by changing the signs of all the terms of the equation when necessary. Then the equations must be arranged according to the magnitudes of the quotients found by dividing the constant term of each equation by the coefficient of p, due regard being had to the signs: That is, the equations must be arranged according to the magnitude of the terms $\frac{181}{-67}$, $\frac{50}{-58}$, $\frac{36}{-51}$, etc. In this manner I procured the system of equations G.

$$\begin{array}{rlcrl} -x(^5) = & 1\cdot p-104 & \qquad & x(^{16})= & 75\cdot p-\ 52 \\ x(^{46})= & 8\cdot p-738 & & x(^6) = & 12\cdot p-\ \ 2 \\ -x(^{49})= & 8\cdot p-370 & & -x(^{30})= & 56\cdot p+\ \ 8 \\ -x(^{51})= & 17\cdot p-683 & & -x(^{53})= & 19\cdot p+\ \ 7 \\ -x(^{50})= & 15\cdot p-510 & & x(^{41})= & 28\cdot p+\ 14 \\ x(^{47})= & 4\cdot p-134 & & x(^{38})= & 30\cdot p+\ 17 \\ x(^{35})= & 18\cdot p-227 & & x(^{39})= & 30\cdot p+\ 44 \\ -x(^{52})= & 21\cdot p-228 & & -x(^{31})= & 48\cdot p+\ 78 \\ x(^{43})= & 21\cdot p-223 & & x(^{37})= & 26\cdot p+\ 51 \\ -x(^{27})= & 101\cdot p-1050 & & -x(^{28})= & 241\cdot p+475 \\ x(^{33})= & 10\cdot p-\ 86 & & x(^{15})= & 83\cdot p+235 \\ x(^{44})= & 15\cdot p-129 & & x(^{11})= & 35\cdot p+116 \\ x(^{14})= & 73\cdot p-518 & & x(^{56})= & 21\cdot p+\ 70 \end{array}$$

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\begin{array}{rlrl}
x(^{21}) = & 35 \cdot p - 244 & x(^{7}) = & 14 \cdot p + 79 \\
x(^{12}) = & 39 \cdot p - 258 & -x(^{26}) = & 88 \cdot p + 522 \\
x(^{17}) = & 80 \cdot p - 493 & -x(^{24}) = & 42 \cdot p + 291 \\
x(^{8}) = & 22 \cdot p - 105 & x(^{40}) = & 29 \cdot p + 203 \\
x(^{13}) = & 42 \cdot p - 148 & x(^{19}) = & 68 \cdot p + 489 \qquad (G) \\
-x(^{1}) = & 67 \cdot p - 181 & -x(^{23}) = & 29 \cdot p + 313 \\
x(^{9}) = & 26 \cdot p - 68 & x(^{18}) = & 73 \cdot p + 835 \\
x(^{20}) = & 49 \cdot p - 103 & x(^{42}) = & 26 \cdot p + 325 \\
x(^{34}) = & 16 \cdot p - 32 & -x(^{48}) = & 7 \cdot p + 102 \\
-x(^{32}) = & 39 \cdot p - 75 & x(^{22}) = & 22 \cdot p + 327 \\
-x(^{25}) = & 55 \cdot p - 72 & -x(^{54}) = & 17 \cdot p + 426 \\
x(^{36}) = & 24 \cdot p - 25 & x(^{45}) = & 16 \cdot p + 481 \\
x(^{10}) = & 32\, p - 30 & -x(^{55}) = & 14 \cdot p + 909 \\
-x(^{4}) = & 49 \cdot p - 45 & & \\
-x(^{2}) = & 58 \cdot p - 50 & & \\
-x(^{29}) = & 66 \cdot p - 51 & & \\
-x(^{3}) = & 51 \cdot p - 36 & &
\end{array}
$$

The sum of all the coefficients of p in these equations is 2211, which put=F. The sum of the first thirty of the coefficients is 1062, which is less than $\frac{1}{2}$ F, and by adding the next coefficient the sum becomes 1137, which is greater than $\frac{1}{2}$ F, hence by the rule given by La Place the value of p, which will render the sum of the errors $x(^{1})$, $x(^{2})$, &c. taken positively a *minimum* will be found by putting the second member of the thirty first of the equations (G) equal to 0; that is, $75 \cdot p - 52 = 0$, which gives $p = 0 \cdot 693$. By substituting this value in the equations marked (4), (3), (2), (1), we successively obtain

$n = 0 \cdot 49828 \cdot p + 0 \cdot 80842 = 1 \cdot 154.$

$d = 0 \cdot 58044 \cdot p + 0 \cdot 91378 \cdot n - 0 \cdot 86191 = 0 \cdot 595.$

$i = -0 \cdot 22944 \cdot d + 0 \cdot 15555 \cdot p + 0 \cdot 06885 \cdot n + 0 \cdot 30130 = 0 \cdot 352.$

$t = -0 \cdot 40480 \cdot d + 0 \cdot 47493 \cdot p + 0 \cdot 24398 \cdot n + 0 \cdot 55366 \cdot i - 0 \cdot 02703 = 0 \cdot 537.$

These values being substituted in the preceding expressions of the elements of the orbit, they become

$D+0{\cdot}002 \cdot d = 0{\cdot}64969$

$T+0{\cdot}2 \cdot t =$ Sep. 18d. 6074

$P+10' \cdot p = 9s\ 1^{\circ}\ 0'\ 11''$

$N-10' \cdot n = 8\ 26\ 24\ 57$

$I-10' \cdot i \quad = 63\ \ 9\ 42$

These would be the true elements depending on the principles laid down if the variations of the calculated geocentric longitudes and latitudes were strictly proportional to the variations of the elements $0{\cdot}002{\cdot}d$, $0{\cdot}2{\cdot}t$, &c. but as this is not the case, it was necessary to repeat in part the operation to make the calculations agree exactly with those principles. For this purpose I made use of the approximate values of the elements last found, instead of D, T, P, N, and I, and recalculated the values of l as in the first supposition, and thus obtained new values of the constant terms of the equations A, instead of +38, —88, etc. These new terms arranged in the order of the equations are 126, 0, —10, 1, 93, —12, 76, —108, —66, —28, 122, —248, —137, —490, 261, —25, —466, 860, 507, —102, —251, 316, —356, —346, 14, —607, 960, —643, 4, —48, —112, 46, —77, —20, —212, —11, 68, 35, 62, 224, 37, 346, —206, —114, 495, —727, —127, —101, 369, 510, 683, 224, —9, —426, —907, —115, and by substituting them in the equations A, and making the corresponding alterations in the constant terms of the equations B, C, D, etc. a new system of equations G was obtained, and by operating on them in the same manner as before I found that the value of p, which would render the sum of the errors $x(^1)$, $x(^2)$, &c. a minimum, would be had by putting $75{\cdot}p+2=0$, whence $p=-0{\cdot}027$, which, substituted in the equations (4), (3), (2), (1), deduced from these last calculations, will give $n=-0{\cdot}011$, $d=-0{\cdot}036$, $i=-0{\cdot}025$, $t=0{\cdot}0013$. Consequently the corrections of the last found elements are $0{\cdot}002{\cdot}d=-0{\cdot}00007$, $0{,}2{\cdot}t=$

0·00026, $10'\cdot p = -16''$, $-10'\cdot n = 6''$, $-10'\cdot i = 15''$. These corrections being applied to those elements produce the following

Correct elements of the orbit of the comet.

Perihelion distance 0·64962. The mean distance of the earth from the sun being denoted by unity.

Time of passing the perihelion September 18,60766, or Sep. 18d. 14h. 35′ 2″ mean time at Salem, corresponding to Sep. 18d. 19h. 18′ 34″ mean time at Greenwich.

Longitude of the Ascending Node	8s 26° 25′ 3″
Place of the Perihelion counted on the orbit of the comet	9 0 59 55
Inclination of the orbit to the ecliptic	63 9 57

Motion direct.

By these elements I calculated the geocentric longitudes and latitudes contained in the following Table.

Mean time of observation at Salem.	Observed longitude.	Calculated longitude.	Difference.		Observed latitude.	Calculated lat.	Difference.	
1807 d h ′ ″	s ° ′ ″	s ° ′ ″	′ ″	′ ″	North.	North.	′ ″	′ ″
Sep. 26·6·32·33	7· 6· 8· 4	7· 6· 8· 3		—0· 1	12·22·13	12·23·27	+1·14	
Oct. 5·8·12·34	7·13·40·36	7·13·40·32		—0· 4	23·44·21	23·41·38		—2·43
8 6·59·22	7·16· 6·58	7·16· 4·41		—2· 17	27· 6·45	27· 6·38		—0· 7
9·6·47·11	7·16·53·36	7·16·53·25		—0·11	28·12·54	28·13·39	+0·45	
10·6·41·21	7.17·42·37	7·17·42·36		—0· 1	29·18·11	29·20 0	+1·49	
11·6·49·54	7·18 32·49	7·18·32·36		—0·13	30·26·52	30·26· 3		—0·49
19·7·25·16	7·25·33· 4	7·25·31·17		—1·47	38·35·58	38·37·15	+1·17	
21·7·15·40	7·27·23·24	7·27·23·21		—0· 3	40·29·18	40·29·38	+0·20	
22·6·54·40	7·28·21·45	7·28·20·14		—1·31	41·20·15	41·23·47	+3·32	
23 6·27·56	7·29·16·29	7·29 18· 2	+1·33		42·16·38	42·16·49	+0·11	
24·6·25·12	8· 0·17· 9	8· 0·17·59	+0·50		43·10·55	43· 9·47		—1· 8
25·7·14·15	8· 1·21·17	8· 1·21·29	+0·12		44· 4·15	44· 3·41		—0·34
26·6·36·35	8· 2·24·51	8· 2·22·32		—2·19	44·54·34	44·53·33		—1· 1
27·6·24· 5	8· 3 22·12	8· 3·26· 3	+3·51		45·47· 2	45·43·19		—3·43
28·6·36·22	8· 4·30·12	8· 4·32· 1	+1·59		46·33·36	46·33· 0		—0·36
		Sums.	+8·25	—8·22		Sums.	+7·54	—7·58
Nov. 6·8·11·31	8 15·36·15	8·15·44· 3	+7·48		53·19·52	53·14· 9		— 5·43
9·6·20·28	8·19·59·16	8·19·54·32		—4· 44	55· 1·14	55· 4·44	+3·30	
10·6·27·37	8·21·24·46	8·21·24·47	+0· 1		55·38·29	55·40·27	+1·58	
11·6·21·16	8·22·49· 0	8·22·56·22	+7·22		56·22·46	56·14·36		— 8·10
14·6·28· 2	8·28· 0·23	8 27·45·38		—14·45	57·37·53	57·50· 5	+12·12	

16·6·11·11	9· 1·16·32	9· 1· 7·39		—8·53	58·44·29	58·46·42	+ 2 13	
20·6·31· 2	9· 8·18·16	9· 8·19·31	+1·15		60·22·12	60·24· 0	+ 1·48	
22·6·59·41	9·12· 4· 0	9·12· 7·44	+3·44		61·10· 0	61· 3·59		— 6· 1
23·8· 6· 5	9·14·14· 6	9·14· 8·23		—5·43	61·30·33	61·22·11		— 8·22
28·8·27·30	9·23·57·23	9·24· 2·53	+5·30		62·37·20	62·26· 7		—11·13
29·6·52·48	9·25·50 10	9·25·55·30	+5·20		62·37·18	62·33·45		— 3·33
30.7·24·27	9·27·59·27	9·27·58·47		—0·40	62·40·10	62·40·30	+ 0·20	
Dec. 3·7·27· 0	10· 3·53·49	10· 4· 3·32	+ 9·43		62·45· 2	62·52·20	+ 7·18	
4·7·13·45	10· 6·18·16	10· 6· 1·53		—16·23	62·37·41	62·53· 0	+15·19	
17·6·26·30	10·29·31·47	10·29·42·16	+10·29		61·12·32	61·11· 4		— 1·28
		Sums.	59·37	—59·30			52·32	—52·28

The first observation in this table was made at Nantucket by Mr. W. Folger jun. the second at Cambridge by Professor Farrar and Mr. I. Nichols, the rest are those I made at Salem by a circle of reflection. The observed and calculated places of the comet in September and October agree in general as well as was to be expected. The disagreement in some of the observations in November and December arose in great measure from the difficulty of observing with a circle, when the comet was very faint, the moon near the full or the weather very damp.

The last time I saw the comet was on the 30th of January, 1808, at 8h. 49′ 10″ mean time at Salem. I estimated roughly its longitude to be about 0s. 15° 12′ and its latitude 47° 22′ N. The longitude calculated by the above elements was 0s. 15° 22′ and the latitude 47° 3′ N. The differences are within the limits of the errors of this observation.

I did not attempt to investigate the elliptical orbit of the comet; for I found that if its mean distance from the sun was sixty times as great as that of the earth, and the other elements the same as in the parabolic orbit, the differences between the heliocentric places, calculated in the elliptical and in the parabolic orbit, from September to the middle of November, rarely exceeded four minutes; so that it would have been

in vain to have attempted to calculate the elliptical motion without observations made to a much greater degree of precision. It is possible that on this account the preceding elements may require some small corrections.

None of the elements of the orbits of the known comets, inserted by De La Lande and Vince in their systems of Astronomy, agree with the orbit of this comet. That seen in the year 1748 agrees more nearly than any one of them ; but there is so great a difference in the longitude of the node and in the inclination of the orbit, that there is not any probability of its being the same comet. We may therefore safely conclude that the comet, whose elements we have here calculated, is one, which was before unknown to Astronomers.